

Cosmological perturbations of Quasi-Dilaton non-linear massive gravity

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We study the cosmological perturbations of the recently proposed extension of non-linear massive gravity with a scalar field. The added scalar field ensures a new symmetry on the field space of the theory. The theory has the property of having a flat dS solution, in contrast to the standard dRGT massive gravity. The tensor part is the same as that of the standard dRGT massive gravity and shows the gravitational wave with time dependent mass which modified the dispersion relation. We obtain the curvature perturbation of the model on superhorizon scales for a specific choice of $\omega = 0$ and find that there is a possibility that the curvature perturbation is damped over such scales.

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I. INTRODUCTION

Gravity, as the oldest force known to man has also had the longest history of trials and tribulations along the road to discovering its nature. Over the course of its development, it has been witnessing a myriad of attempts to unlock its notoriously difficult and mysterious behavior from the largest to the smallest of distances. The challenge has been truly spectacular. Since the first formulation of the gravitational field by Newton and centuries later by Einstein in the form of the theory of general relativity (GR), the scenery is still cluttered with debris left from various attempts to understand its hard to grasp nature. Even today, the challenge is as fresh and as interesting as ever. Not surprisingly, building on GR, the last couple of decades have been particularly rich in new ideas and approaches which attempt to formulate the gravitational field in such a way as to pave the way to a formulation of the theory which would explain such recently observed phenomenon as the accelerated expansion of the universe, galaxy rotation curves and even the birth of the universe. One such attempt has been surfacing over the past few years in the form of what is now known as massive gravity which, as the name suggests, is a theory with a massive graviton as the building block of the gravitational field.

The notion of a massive graviton has been a tempting and challenging premise in theoretical physics. One of the main motivations of having a massive graviton is that gravity could become weak at large distances, thus mimicking the effect of accelerated expanding universe. The problem is not as easy as it seems. The first attempt to build a theory for a massive spin-2 field was back in 1939 when Fierz and Pauli (FP) developed a linear theory for a massive graviton [1]. It took thirty years for physicists to find out that the theory does not reduce to the standard GR when one takes the limit $m \rightarrow 0$ [2]. The problem was soon addressed by Vainshtein [3] who proposed that adding non-linearities to the action can cure the problem and screen the effect of helicity-0 component of the massive graviton at solar system scales. The simplest possible non-linearity one can add to the FP action is by replacing the linear kinetic term for the helicity-2 field with the fully non-linear, and still ghost free, Einstein-Hilbert action. However, the resulting action have proven to have a ghost instability which was discovered by Boulware and Deser [4]. The problem arises because the lapse function is no longer a Lagrange multiplier. This new problem can then be solved if one appropriately adds interaction terms to the Lagrangian and again makes the lapse function a Lagrange multiplier order by order in non-linearities [5, 6].

In this paper, after a brief review of the theory we study the cosmological perturbations around an accelerating solution and obtain the background equations. The second order Lagrangian can then be obtained using the perturbed FRW metric. It is an immediate observation from the form of the second order Lagrangian that the tensor, vector and scalar modes do not couple. The tensor mode shows a massive gravitational wave with a time-dependent mass parameter which we shall obtain in section IV A. We show in section IV B that the vector mode vanishes when one integrates out the non-dynamical vector field and then obtain the scalar mode in section IV C. Two of the scalar modes are non-dynamical and can be integrated out immediately with three degrees of freedom remaining in the scalar sector. We show that in the superhorizon limit where one of the degrees of freedom does not play a role, the

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curvature perturbation can be obtained analytically, and there is a vast region in the parameter space over which the curvature perturbation will damp on the superhorizon scales.

II. A BRIEF REVIEW

Recently, a two parameter theory of massive gravity has been proposed [7]. With the aid of a new re-summation of the non-linear interaction [8] de Rham, Gabadadze and Tolley (dRGT) have constructed a theory which is free of ghost instabilities in the decoupling limit. The theory has been proven to be ghost-free in the full non-linear theory and hence a reliable effective field theory of a massive spin-2 field [9]. The action for dRGT non-linear massive gravity can be written as

$$\mathcal{L} = \frac{M_{pl}^2}{2} \sqrt{-g} \left(R + m^2 \mathcal{U}(\mathcal{K}) \right) + \sqrt{-g} \mathcal{L}_m(g_{\mu\nu}, \psi), \quad (1)$$

where the non-linear interactions are collected in \mathcal{U}

$$\mathcal{U}(\mathcal{K}) = \mathcal{U}_2 + \alpha_3 \mathcal{U}_3 + \alpha_4 \mathcal{U}_4, \quad (2)$$

which consists of polynomials of various traces of the matrix $\mathcal{K}_\nu^\mu(g, \phi^a) = \delta_\nu^\mu - \sqrt{g^{\mu\alpha} f_{\alpha\nu}}$ where the fiducial metric is defined as $f_{\alpha\nu} = \partial_\alpha \phi^a \partial_\nu \phi^b \eta_{ab}$ and ϕ^a are the Stuckelberg fields responsible for the breaking of general covariance and the rectangular brackets denote traces, $[\mathcal{K}] \equiv \text{Tr}(\mathcal{K}) = \mathcal{K}_\mu^\mu$,

$$\mathcal{U}_2 = [\mathcal{K}]^2 - [\mathcal{K}^2], \quad (3a)$$

$$\mathcal{U}_3 = [\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3], \quad (3b)$$

$$\mathcal{U}_4 = [\mathcal{K}]^4 - 6[\mathcal{K}^2][\mathcal{K}]^2 + 8[\mathcal{K}^3][\mathcal{K}] + 3[\mathcal{K}^2]^2 - 6[\mathcal{K}^4]. \quad (3c)$$

The first term coincides with the Fierz-Pauli mass term at the linear level, and the last two terms are non-linear interactions which ensure that the theory has no ghost.

One of the interesting property of the theory is that if one assumes the Stuckelberg fields to be in the unitary gauge defined as $\phi^a = \delta_\mu^a x^\mu$, the theory does not have a non-trivial flat FRW solution [10]. However, the theory has an open FRW solution with an additional consideration in that one must transform the field space to the open slicing of the Minkowski metric [11]. The cosmological perturbations around such a solution was also considered in [12] where the authors found that the scalar, vector and tensor modes actually decouple and as a result the vector mode does not play any role in the theory. The tensor mode describes a massive gravitational wave with a time-dependent mass. Cosmological evidences of dRGT theory is also considered in [13]. For a review on the theoretical aspect of the theory see [14].

One can also let the non-dynamical metric of the theory to have a kinetic term and hence construct a bimetric theory [15] which has been proven to be ghost free [16]. The cosmological aspects of such bimetric theory is investigated in [17]. One may continue the procedure to build a multimetric gravity theory with dRGT non-linear mass terms. In [18], the authors show that the theory is ghost free in the metric formulation only when the interactions between gravitons are not cyclic. However, in [19] the authors show that in the vielbein formulation of the theory any interactions are allowed. Also in [20] the authors show that the problem of having no flat FRW solution persists in multimetric theories. In fact if one of the metrics is assumed to be flat, all of the other metrics will become flat.

One of the solutions to the problem of the non-existence of the flat FRW solution in the theory is by extending the theory in such a way that the graviton mass becomes a function of some scalar field φ [21]. The cosmological solutions and dynamical analysis of such theories are considered in [22].

Another way to extend the theory is to couple a scalar field to the mass Lagrangian such that the resulting new Lagrangian has an extra symmetry. In particular, one can couple the scalar field to the Lagrangian to achieve dilation invariance on the field space [23]

$$\sigma \rightarrow \sigma - M_{pl} \alpha, \quad \phi^a \rightarrow e^\alpha \phi^a. \quad (4)$$

In the Einstein frame one can write the new action as

$$\mathcal{L} = \frac{M_{pl}^2}{2} \sqrt{-g} \left[R - \frac{\omega}{M_{pl}^2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma + m^2 \mathcal{U}(\tilde{\mathcal{K}}) \right] + \sqrt{-g} \mathcal{L}_m(g_{\mu\nu}, \psi), \quad (5)$$

where $\tilde{\mathcal{K}}_\nu^\mu$ is now defined as

$$\tilde{\mathcal{K}}_\nu^\mu(g, \phi^a) = \delta_\nu^\mu - e^{\sigma/M_{pl}} \sqrt{g^{\mu\alpha} f_{\alpha\nu}} \quad (6)$$

Only the purely geometrical part of the above action is invariant under transformation (4), thus the acronym Quasi-Dilaton (QD) for the scalar field [23]. This theory has been proven to be free of ghost in the Minkowski background if $\omega > 6$. The most interesting feature of this theory is that it admits a flat de Sitter solution even if the Stuckelberg fields are in the unitary gauge [23]. The de Sitter solution is also stable in the decoupling limit for

$$\alpha_3 \neq 0, \quad 0 < \alpha_4 < \frac{\alpha_3^2}{8}, \quad 0 \leq \omega < 6. \quad (7)$$

III. THE BACKGROUND EQUATION

Let us assume that the background metric is of the form

$$ds^2 = -N(t)^2 dt^2 + a(t)^2 (dx^2 + dy^2 + dz^2), \quad (8)$$

and the Stuckelberg fields take the form

$$\phi^0 = f(t), \quad \phi^i = \delta_\mu^i x^\mu.$$

Note that we will work in the unitary gauge. However, in order to obtain the Stuckelberg equation from the action we assume the above form for the Stuckelberg fields, and finally set $f(t) = t$. We also assume that the QD field only depends on t . By varying the action (1) with respect to $f(t)$, one obtains the constraint equation as

$$-9m^2 M_{pl}^2 e^{\sigma/M_{pl}} \left(a - e^{\sigma/M_{pl}} \right) \left[\frac{4}{3} \alpha_4 e^{2\sigma/M_{pl}} - \left(\frac{8}{3} \alpha_4 - \alpha_3 \right) a e^{\sigma/M_{pl}} + \left(\frac{4}{3} \alpha_4 + \alpha_3 + \frac{1}{3} \right) a^2 \right] = k_1. \quad (9)$$

We are interested to the set of equations for which $k_1 = 0$. In this case one can then solve the above equation by using the ansatz

$$e^{\sigma/M_{pl}} = Xa,$$

in the equation which results in

$$X = \frac{3\alpha_3 + 8\alpha_4 \pm \sqrt{9\alpha_3^2 - 16\alpha_4}}{8\alpha_4}, \quad X = 1. \quad (10)$$

The solution $X = 1$ is not acceptable because in this case the effective cosmological constant vanishes, and the consistency of the theory does not allow one to have a flat cosmological solution [23]. Putting the above ansatz in the mass part of the Lagrangian, one can write the Friedman and Raychaudhuri equations as

$$3H^2 - \frac{\omega}{M_{pl}^2} \frac{\dot{\sigma}^2}{2N^2} = 3M_f, \quad (11)$$

$$2\frac{\dot{H}}{N} + 3H^2 + \frac{\omega}{M_{pl}^2} \frac{\dot{\sigma}^2}{2N^2} = 3M_g. \quad (12)$$

where we define $H = \dot{a}/(aN)$ and

$$M_f = \frac{m^2}{16\alpha_4^2} \left[3\alpha_3^2 (3(X-1)\alpha_3 - 1) + 4(1 - 4(X-1)\alpha_3) \alpha_4 \right], \quad (13)$$

$$M_g = \frac{m^2}{3} \left[-6 - X(-6 - 3r + X + 2rX) + 3(X-1)(4 + (-2 + r(X-3))X) \alpha_3 + 12(X-1)^2(rX - 1) \alpha_4 \right]. \quad (14)$$

and $r \equiv r(t) = a(t)/N(t)$. We will use these forms of the background equations in the second order Lagrangian. Substituting equation (10) in (11) leads to a constant Hubble parameter, showing a de Sitter solution

$$H^2 \equiv H_0^2 = \left(\frac{6m^2(1-X)[2 + 4\alpha_3 + 4\alpha_4 + X(-1 + (X-5)\alpha_3 + 4(X-2)\alpha_4)]}{\omega - 6} \right)^{1/2}. \quad (15)$$

From equation (12) we obtain that r should be a constant given by

$$r = \frac{12(1 + 2\alpha_3 + 2\alpha_4)\omega + 4X^2(1 + 6\alpha_3 + 12\alpha_4)(3 + \omega) - 3X^3(\alpha_3 + 4\alpha_4)(6 + \omega) - 3X(1 + 3\alpha_3 + 4\alpha_4)(6 + 5\omega)}{X(\omega - 6)(3 + 9\alpha_3 + 12\alpha_4 + 3X^2(\alpha_3 + 4\alpha_4) - 2X(1 + 6\alpha_3 + 12\alpha_4))}. \quad (16)$$

The scalar field equation is satisfied automatically by plugging in equations (10), (15) and (16).

IV. SECOND ORDER LAGRANGIAN

In this section we study perturbations signified by

$$ds^2 = -N(t)^2[1 + 2\phi(t, x, y, z)]dt^2 + 2N(t)a(t)\beta_i(t, x, y, z)dtdx^i + a(t)^2[\delta_{ij} + h_{ij}(t, x, y, z)]dx^i dx^j, \quad (17)$$

where ϕ , β_i and h_{ij} are the perturbation variables of the FRW metric. The perturbation of the Stuckelberg fields around unitary gauge is

$$\phi^a = x^a + \pi^a(t, x, y, z), \quad (18)$$

and the perturbation of the dilaton field around the background solution has the form

$$\sigma = \sigma_0(t) + \zeta(t, x, y, z). \quad (19)$$

Now, we consider the infinitesimal coordinate transformation

$$x^\mu \rightarrow x^\mu + \xi^\mu(t, x, y, z), \quad (20)$$

which leads to the change of the perturbed quantities as

$$\phi \rightarrow \phi - \frac{1}{N}\partial_t(N\xi^0), \quad (21a)$$

$$\beta_i \rightarrow \beta_i + \frac{N}{a}\partial_i\xi^0 - \frac{a}{N}\dot{\xi}_i, \quad (21b)$$

$$h_{ij} \rightarrow h_{ij} - \partial_i\xi_j - \partial_j\xi_i - 2NH\xi^0\delta_{ij}, \quad (21c)$$

$$\pi^a \rightarrow \pi^a - \xi^a, \quad (21d)$$

$$\zeta \rightarrow \zeta - \dot{\sigma}_0\xi^0, \quad (21e)$$

where prime denotes the time derivation.

Using the perturbations of the Stuckelberg fields one can construct the gauge invariant quantities using the perturbed Stuckelberg fields in the following manner

$$\Phi = \phi - \frac{1}{N}\partial_t(N\pi^0), \quad (22a)$$

$$\mathcal{B}_i = \beta_i + \frac{N}{a}\partial_i\pi^0 - \frac{a}{N}\dot{\pi}_i, \quad (22b)$$

$$\mathcal{H}_{ij} = h_{ij} - \partial_i\pi_j - \partial_j\pi_i - 2NH\pi^0\delta_{ij}, \quad (22c)$$

$$\mathcal{Z} = \zeta - \dot{\sigma}_0\pi^0. \quad (22d)$$

We may decompose the gauge invariant vector and tensor parts as

$$\mathcal{B}_i = \partial_i\beta + S_i, \quad (23a)$$

$$\mathcal{H}_{ij} = 2\psi\delta_{ij} + \partial_i\partial_j E + \frac{1}{2}(\partial_i F_j + \partial_j F_i) + \gamma_{ij}, \quad (23b)$$

where

$$\begin{aligned} \partial^i S_i &= 0 = \partial^i F_i, \\ \partial^i \gamma_{ij} &= 0 = \delta^{ij} \gamma_{ij}. \end{aligned} \quad (24)$$

We note that there are four degrees of gauge freedom because of the coordinate transformation, two of which represent the scalar part and the others relate to the vector part. One may fix the gauge freedom by the choice

$$\pi^0 = 0, \quad \pi^i = 0. \quad (25)$$

Note that with this gauge fixing all the gauge invariant perturbation variables become equal to the original one, making our calculations simpler. Also note that the above gauge fixing is similar to the use of the unitary gauge, and so the fiducial metric takes the form

$$f_{\mu\nu} = \eta_{\mu\nu}. \quad (26)$$

The components of the $f_\nu^\mu = g^{\mu\rho} f_{\rho\nu}$ matrix in the unitary gauge are

$$f_0^0 = \frac{1}{N^2} (1 - 2\phi + 4\phi^2 - \beta_i \beta^i) + \mathcal{O}(\epsilon^3), \quad (27a)$$

$$f_i^0 = \frac{1}{Na} (\beta_i - \beta^j h_{ji} - 2\phi \beta_i) + \mathcal{O}(\epsilon^3), \quad (27b)$$

$$f_0^i = -\frac{1}{Na} (\beta^i - \beta_j h^{ji} - 2\phi \beta^i) + \mathcal{O}(\epsilon^3), \quad (27c)$$

$$f_j^i = \frac{1}{a^2} (\delta_j^i - h_j^i - \beta^i \beta_j + h^{ik} h_{kj}) + \mathcal{O}(\epsilon^3). \quad (27d)$$

To compute the components of the $\tilde{\mathcal{K}}_\nu^\mu$, we use the method presented in [12] to expand the square root in (6).

To zeroth order perturbation we find

$$\tilde{\mathcal{K}}^{(0)0}_0 = 1 - \frac{\Delta}{N}, \quad \tilde{\mathcal{K}}^{(0)i}_0 = 0 = \tilde{\mathcal{K}}^{(0)0}_i, \quad \tilde{\mathcal{K}}^{(0)i}_j = \left(1 - \frac{\Delta}{a}\right) \delta_j^i, \quad (28)$$

where $\Delta = e^{\sigma_0/M_{pl}}$. The first and second orders are

$$\tilde{\mathcal{K}}^{(1)0}_0 = \frac{\Delta}{N} \left(\phi - \frac{\zeta}{M_{pl}} \right), \quad \tilde{\mathcal{K}}^{(1)0}_i = -\frac{\Delta}{N(1+r)} \beta_i, \quad \tilde{\mathcal{K}}^{(1)i}_0 = \frac{\Delta}{N(1+r)} \beta^i, \quad \tilde{\mathcal{K}}^{(1)i}_j = \frac{\Delta}{a} \left(\frac{1}{2} h_j^i - \frac{\zeta}{M_{pl}} \delta_j^i \right), \quad (29)$$

$$\tilde{\mathcal{K}}^{(2)0}_0 = \frac{\Delta}{N} \left(\frac{r(r+2)}{2(r+1)^2} \beta^i \beta_i - \frac{3}{2} \phi^2 + \frac{1}{M_{pl}} \zeta \phi - \frac{1}{2M_{pl}^2} \zeta^2 \right), \quad (30a)$$

$$\tilde{\mathcal{K}}^{(2)0}_i = \frac{\Delta}{N(r+1)} \left(\frac{r+2}{r+1} \phi \beta_i + \frac{2r+1}{2(r+1)} \beta^j h_{ji} - \frac{1}{M_{pl}} \zeta \beta_i \right), \quad (30b)$$

$$\tilde{\mathcal{K}}^{(2)i}_0 = -\frac{\Delta}{N(r+1)} \left(\frac{r+2}{r+1} \phi \beta^i + \frac{2r+1}{2(r+1)} \beta_j h^{ji} - \frac{1}{M_{pl}} \zeta \beta^i \right), \quad (30c)$$

$$\tilde{\mathcal{K}}^{(2)i}_j = \frac{\Delta}{2a} \left(\frac{2r+1}{(r+1)^2} \beta^i \beta_j - \frac{3}{4} h^{ik} h_{kj} + \frac{1}{M_{pl}} \zeta h_j^i - \frac{1}{M_{pl}^2} \zeta^2 \delta_j^i \right), \quad (30d)$$

where $r = \frac{a}{N}$, as that in the background. The traces of $\tilde{\mathcal{K}}$ for zero and first order perturbation are given by

$$[\tilde{\mathcal{K}}^n]^{(0)} = 3(1-X)^n + (1-rX)^n, \quad (31)$$

$$[\tilde{\mathcal{K}}^n]^{(1)} = nrX(1-rX)^{n-1} \left(\phi - \frac{1}{M_{pl}} \zeta \right) + \frac{n}{2} X(1-X)^{n-1} \left(h - \frac{6}{M_{pl}} \zeta \right), \quad (32)$$

where $X = \Delta/a$. To second order we obtain

$$[\tilde{\mathcal{K}}]^{(2)} = \frac{r_2}{2r_1} X \beta^i \beta_i - \frac{3}{8} X h^{ij} h_{ij} - \frac{1}{2} r X \left(3\phi^2 + \frac{1}{M_{pl}^2} \zeta^2 \right) + \frac{1}{2M_{pl}} X \zeta (h + 2\phi) - \frac{3}{2M_{pl}^2} X \zeta^2, \quad (33a)$$

$$[\tilde{\mathcal{K}}^2]^{(2)} = \frac{r_2 - X r_3}{r_1} X \beta^i \beta_i + (4rX - 3) X r \phi^2 + \frac{2}{M_{pl}} (1 - 2rX) X r \phi \zeta + \frac{1}{M_{pl}^2} [3(2X - 1) + r(2rX - 1)] X \zeta^2 \\ + \frac{1}{M_{pl}} (1 - 2X) X h \zeta + (X - \frac{3}{4}) X h^{ij} h_{ij}, \quad (33b)$$

$$[\tilde{\mathcal{K}}^3]^{(2)} = \frac{3}{2r_1} (r_2 - 2r_3 X + r_4 X^2) X \beta^i \beta_i - \frac{3}{8} (3 - 5X) (1 - X) X h_{ij} h^{ij} - \frac{3}{2} (1 - rX) (3 - 5rX) X r \phi^2 \\ + \frac{3}{M_{pl}} (1 - rX) (1 - 3rX) X r \phi \zeta + \frac{3}{2M_{pl}} (1 - X) (1 - 3X) X h \zeta \\ - \frac{3}{2M_{pl}^2} [(r + 3) - 4X(r^2 + 3) + 3X^2(r^3 + 3)] X \zeta^2, \quad (33c)$$

$$[\tilde{\mathcal{K}}^4]^{(2)} = \frac{2}{r_1} (r_2 - 3X r_3 + 3X^2 r_4 - X^3 r_5) X \beta_i \beta^i + \frac{3}{2} (2X - 1) (1 - X)^2 X h_{ij} h^{ij} + 6(2rX - 1) (1 - rX)^2 X r \phi^2 \\ + \frac{4}{M_{pl}} (1 - 4rX) (1 - rX)^2 X r \phi \zeta + \frac{2}{M_{pl}^2} (-r - 3 + 6X(r^2 + 3) - 9X^2(r^3 + 3) + 4X^3(r^4 + 3)) X \zeta^2 \\ + \frac{2}{M_{pl}} (1 - 4X) (1 - X)^2 X h \zeta. \quad (33d)$$

where

$$r_n = \sum_{i=0}^n r^i. \quad (34)$$

From the above formulae, one can construct the mass term using equations (3). The gauge invariant second order Lagrangian can then be written as

$$S^{(2)} = M_{pl}^2 \int d^4x N a^3 \left(\mathcal{L} + \frac{3}{2} M_f (-\Phi^2 + \mathcal{B}^i \mathcal{B}_i + \Phi \mathcal{H}) + \frac{3}{8} (2M_f - M_g) (\mathcal{H}^2 - 2\mathcal{H}_{ij} \mathcal{H}^{ij}) + \mathcal{L}_{mass} \right), \quad (35)$$

where we have defined

$$\mathcal{L} = \frac{1}{8N^2} (\dot{\mathcal{H}}_{ij} \dot{\mathcal{H}}^{ij} - \dot{\mathcal{H}}^2) + \frac{H}{N} \Phi \dot{\mathcal{H}} - \frac{1}{a} \left(2H\Phi - \frac{1}{2N} \dot{\mathcal{H}} \right) \partial_i \mathcal{B}^i - \frac{1}{2Na} \partial_i \mathcal{B}_j \dot{\mathcal{H}}^{ij} - 3H^2 \Phi^2 + \frac{1}{4a^2} (\partial_i \mathcal{B}_j \partial^i \mathcal{B}^j - (\partial_i \mathcal{B}^i)^2) \\ + \frac{1}{2a^2} (\partial_i \partial_j \mathcal{H}^{ij} - \nabla^2 \mathcal{H}) \Phi + \frac{1}{8a^2} (2\partial^i \mathcal{H}_{ik} \partial_j \mathcal{H}^{jk} + \mathcal{H}_{ij} \nabla^2 \mathcal{H}^{ij} + 2\mathcal{H} \partial_i \partial_j \mathcal{H}^{ij} - \mathcal{H} \nabla^2 \mathcal{H}) \\ + \frac{\omega}{M_{pl}^2} \left(\frac{\mathcal{Z}^2}{2N^2} + \frac{\dot{\sigma}_0}{2N^2} (\mathcal{H} - 2\Phi) \dot{\mathcal{Z}} + \frac{1}{2a^2} \mathcal{Z} \nabla^2 \mathcal{Z} + \frac{\dot{\sigma}_0}{aN} \mathcal{Z} \partial_i \mathcal{B}^i + \frac{\dot{\sigma}_0^2}{2N^2} \Phi^2 \right), \quad (36)$$

and

$$\mathcal{L}_{mass} = M_1 \mathcal{H}_{ij} \mathcal{H}^{ij} + M_2 \mathcal{H}^2 + M_\zeta \mathcal{Z}^2 + (M_{h\zeta} \mathcal{H} + M_{\zeta\phi} \Phi) \mathcal{Z} + M_\phi \Phi^2 + M_\beta \mathcal{B}_i \mathcal{B}^i + M_{h\phi} \mathcal{H} \Phi. \quad (37)$$

The definition of M_i 's are given in Appendix A. We have also used equations (11) and (12) to simplify the action. The above action ensures that the scalar, vector and tensor modes do not couple to each other. Therefore we study them separately.

A. Tensor mode

Keeping only γ_{ij} in the action (35), we obtain

$$S_{tensor}^{(2)} = M_{pl}^2 \int d^4x N a^3 \left[\frac{1}{8N^2} \dot{\gamma}_{ij} \dot{\gamma}^{ij} + \frac{1}{8a^2} \gamma_{ij} \nabla^2 \gamma^{ij} - \frac{1}{4} (3M_g - 4M_1) \gamma_{ij} \gamma^{ij} \right]. \quad (38)$$

Variation of the above action with respect to γ_{ij} leads to the equation of motion for tensor perturbations

$$\frac{\partial}{\partial t} \left(\frac{a^3}{N} \dot{\gamma}^{ij} \right) - Na \nabla^2 \gamma^{ij} + 2(3M_g - 4M_1)Na^3 \gamma^{ij} = 0. \quad (39)$$

Fourier transforming the equation and using the conformal time define as

$$d\eta = \frac{N}{a} dt, \quad (40)$$

one can write equation (39) as

$$\bar{\gamma}'' + \left[\vec{k}^2 - \frac{a''}{a} + 2a^2(3M_g - 4M_1) \right] \bar{\gamma} = 0, \quad (41)$$

where we drop the indices of γ_{ij} and define $\bar{\gamma} = \frac{a}{2}\gamma$. This equation shows that the graviton acquires a time-dependent mass in this background. This is in agreement with the result of [12] with a different mass parameter.

B. Vector mode

We now study the vector mode of the action (35). There are two vector modes S^i and F^i in the action. One can write the vector part of the action as

$$S_{vector}^{(2)} = M_{pl} \int d^4x Na^3 \left[-\frac{1}{16N^2} \dot{F}_i \nabla^2 \dot{F}^i + \frac{1}{4Na} \dot{F}_i \nabla^2 S^i - \frac{1}{4a^2} S_i \nabla^2 S^i + \frac{1}{2}(3M_f + 2M_\beta) S_i S^i + \frac{1}{8}(3M_g - 4M_1) F_i \nabla^2 F^i \right]. \quad (42)$$

Varying the action with respect to S^i gives

$$\frac{1}{4Na} \nabla^2 \dot{F}_i - \frac{1}{2a^2} \nabla^2 S_i + (3M_f + 2M_\beta) S_i = 0. \quad (43)$$

By inserting $\nabla^2 S_i$ to the action one obtains

$$S_{vector}^{(2)} = \frac{M_{pl}}{8} (3M_g - 4M_1) \int d^4x Na^3 F_i \nabla^2 F^i, \quad (44)$$

which gives $F_i = 0$ and then $S_i = 0$. So the vector mode of the action vanishes.

C. Scalar mode

In this section we study the scalar perturbations of action (35). We are interested in the behavior of perturbed variables at the superhorizon scales. The scalar part of the second order Lagrangian can be written as

$$\begin{aligned} \mathcal{L}_{scalar} = & \frac{12M_{pl}^2 a}{N} \left[\frac{1}{2} \left(M_1 + M_2 - \frac{3}{4}M_f + \frac{3}{8}M_g \right) N^2 a^2 (\nabla^2 E)^2 + \frac{1}{12} N^2 \left(M_{h\zeta} a^2 \mathcal{Z} - \frac{3}{4} \nabla^2 \Phi \right) \nabla^2 E \right. \\ & + \frac{1}{16} N^2 \psi \nabla^2 (\nabla^2 E) + \frac{1}{8} \left(M_f + \frac{2}{3} M_{h\Phi} \right) N^2 a^2 \Phi \nabla^2 E + \left(\frac{1}{3} M_1 + M_2 + \frac{1}{4} M_f - \frac{1}{8} M_g \right) N^2 a^2 \psi (3\psi + \nabla^2 E) \\ & + \frac{1}{12} N a^2 H \Phi \nabla^2 \dot{E} + \frac{1}{2} a \left(a N H \Phi + \frac{1}{3} N \nabla^2 \beta - \frac{1}{6} a \nabla^2 \dot{E} \right) \dot{\psi} - \frac{1}{4} a^2 \dot{\psi}^2 + \frac{3}{4} \left(M_f + \frac{2}{3} M_{h\Phi} \right) N^2 a^2 \Phi \psi \\ & - \frac{1}{6} N^2 \Phi \nabla^2 \psi - \frac{1}{8} \left(2H^2 - \frac{2}{3} M_\Phi + M_f \right) a^2 N^2 \Phi^2 - \frac{1}{6} N^2 a H \Phi \nabla^2 \beta + \frac{1}{12} M_{\zeta\Phi} N^2 a^2 \mathcal{Z} \Phi \\ & + \frac{1}{2} M_{h\zeta} N^2 a^2 \mathcal{Z} \psi - \frac{1}{12} N^2 \psi \nabla^2 \psi + \frac{1}{12} M_\zeta N^2 a^2 \mathcal{Z}^2 - \frac{1}{12} \left(M_\beta + \frac{3}{2} M_f \right) N^2 a^2 \beta \nabla^2 \beta \\ & \left. + \frac{\omega}{12M_{pl}^2} \left(\frac{1}{2} a^2 \dot{\mathcal{Z}}^2 - a^2 \dot{\sigma}_0 (\Phi - 3\psi - \frac{1}{2} \nabla^2 E) \dot{\mathcal{Z}} + \frac{1}{2} a^2 \dot{\sigma}_0^2 \Phi^2 + \frac{1}{2} N^2 \mathcal{Z} \nabla^2 \mathcal{Z} + \dot{\sigma}_0 N a \mathcal{Z} \nabla^2 \beta \right) \right]. \quad (45) \end{aligned}$$

One can see from equation above that β and Φ are non-dynamical. Transforming back to Fourier space, one can find their equations of motion

$$\beta = \frac{-2M^2HN\Phi + \omega\mathcal{Z}\dot{\sigma} + 2M^2\dot{\psi}}{M^2(2M_\beta + 3M_f)aN}, \quad (46)$$

and

$$\begin{aligned} \Phi = \frac{1}{\Lambda_\phi} \left[\omega \left(k^2 NH\mathcal{Z} - a^2(6M_f + 4M_\beta)\dot{\mathcal{Z}} \right) \dot{\sigma} - 6M^2NH \left(\frac{4}{3}k^2 - 6M_f - 4M_\beta \right) \dot{\psi} - M^2N \left(k^2 a^2 H \dot{E} \right. \right. \\ \left. \left. + a^2N \left(\frac{3}{2}M_f + M_{h\phi} \right) (k^2 E - 6\psi) + a^2M_{\zeta\phi}N\zeta + 2k^2N\psi \right) \right], \end{aligned} \quad (47)$$

where we have defined

$$\Lambda_\phi = a^2M^2N^2(6M_f + 4M_\beta)(3M_f - 2M_\phi + 6H^2) + 8k^2M^2N^2H^2 - a^2\omega(6M_f + 4M_\beta)\dot{\sigma}. \quad (48)$$

At this point we are interested to the behavior of the fields on the superhorizon scales where $k^2 \rightarrow 0$. After substitution of β and Φ from equations (46) and (47) and defining the curvature perturbation as

$$\mathcal{R} = \psi + \frac{H}{\dot{\sigma}_0}\mathcal{Z}, \quad (49)$$

the Lagrangian (45) at the superhorizon scales takes the form

$$\begin{aligned} \mathcal{L}_{scalar}^{k \rightarrow 0} = & - \frac{6M_{pl}^2a^2}{r^3[2M_\phi - 3M_f + (\omega - 6)H_0^2]} \left(\frac{1}{2}r^2(\lambda_5a^2 + 2r\omega H_0^2a + r^2(2M_\phi - 3M_f + \omega H_0^2))\dot{\psi}^2 \right. \\ & - r^2(r\omega H_0^2 + \lambda_5a)a\dot{\mathcal{R}}\dot{\psi} + \frac{1}{2}r^2\lambda_5a^2\dot{\mathcal{R}}^2 - \frac{1}{6}\lambda_3a^4(\mathcal{R} - \psi)^2 - r\lambda_2a^3\psi(\mathcal{R} - \psi) - 2r^2\lambda_1a^2\psi^2 \\ & + H_0r \left[\frac{1}{6}\lambda_6a^2(\mathcal{R} - \psi) + ra[\lambda_4\psi + (M_{pl}M_{\phi\zeta} - \omega H_0^2)\mathcal{R}] + 9r^2 \left(\frac{2}{3}M_{h\phi} + M_f \right) \right] a\dot{\psi} \\ & \left. - \frac{1}{2}H_0r \left[\frac{1}{3}\lambda_6a(\mathcal{R} - \psi) + \omega r[(\omega - 6)H_0^2 + 2M_\phi + 2M_{h\phi}]\psi \right] a^2\psi \right), \end{aligned} \quad (50)$$

where we define

$$\begin{aligned} \lambda_1 = & \left(\frac{3}{4}M_f - \frac{3}{8}M_g + M_1 + 3M_2 \right) (\omega - 6)H_0^2 - \frac{45}{8}M_f^2 - \left(3M_1 + \frac{9}{2}M_{h\phi} + 9M_2 - \frac{9}{8}M_g - \frac{3}{2}M_\phi \right) M_f \\ & + \left(6M_2 - \frac{3}{4}M_g + 2M_1 \right) M_\phi - \frac{3}{2}M_{h\phi}^2, \end{aligned} \quad (51)$$

$$\begin{aligned} \lambda_2 = & \frac{1}{2}\omega(\omega - 6)H_0^2 + \left((M_{pl}M_{h\zeta} + M_\phi + M_{h\phi})\omega - 6M_{pl}M_{h\phi} \right) H_0^2 \\ & - 3M_{pl} \left((M_{h\zeta} + \frac{1}{2}M_{h\phi})M_f + \frac{1}{3}M_{h\phi}M_{\zeta\phi} - \frac{2}{3}M_\phi M_{h\zeta} \right), \end{aligned} \quad (52)$$

$$\lambda_3 = -3\omega H_0^4 + ((M_\phi + M_{pl}^2M_\zeta + M_{pl}M_{\zeta\phi})\omega - 6M_{pl}^2M_\zeta)H_0^2 - 3M_{pl}^2 \left(\frac{1}{6}m_{\zeta\phi}^2 + M_\zeta M_f - \frac{2}{3}M_\phi m_\zeta \right), \quad (53)$$

$$\lambda_4 = \frac{1}{2}\omega(\omega - 4)H_0^2 + (M_{h\phi} + M_\phi)\omega - M_{pl}M_{\zeta\phi}, \quad (54)$$

$$\lambda_5 = \left(H_0^2 - \frac{1}{3}M_\phi + \frac{1}{2}M_f \right) \omega, \quad (55)$$

$$\lambda_6 = -6\lambda_5 + \omega M_{pl}M_{\zeta\phi}. \quad (56)$$

For $\omega = 0$, the field equations at the superhorizon scales are simplified to

$$3r^2H_0MM_{\zeta\phi}\dot{\psi} - a^2\lambda_3(\mathcal{R} - \psi) - 3ra\lambda_2\psi = 0, \quad (57)$$

$$r^3(2M_\phi - 3M_f)(r\ddot{\psi} + 2aH_0\dot{\psi}) + a^2 \left[r^2 H_0 M M_{\zeta\phi} \dot{\mathcal{R}} - \frac{1}{3} \lambda_3 a^2 (\mathcal{R} - \psi) + 2ra(2\lambda_4 H_0^2 - \lambda_2) \psi + ra(4H_0 M M_{\zeta\phi} + \lambda_2) \mathcal{R} + r^2 (9H_0^2(3M_f + 2M_{h\phi}) + 4\lambda_1) \psi \right] = 0, \quad (58)$$

which can be analytically solved for \mathcal{R} and ψ . Substituting \mathcal{R} from (57) into the second equation and solving the resulting equation for ψ results in

$$\psi = t^{\frac{3}{2}} \left(C_1 t^{\frac{\sqrt{9A^2 H_0^2 - 32AB}}{2AH_0}} + C_2 t^{\frac{-\sqrt{9A^2 H_0^2 - 32AB}}{2AH_0}} \right), \quad (59)$$

where C_1 and C_2 are integration constants and we have defined

$$A = M_f M_\zeta - \frac{2}{3} M_\phi M_\zeta + \frac{1}{6} M_{\zeta\phi}^2, \quad (60)$$

$$B = \frac{1}{8} \left(8M_1 + 24M_h - 6M_{h\phi} - 3M_g - 3M_f + 3(M_{\zeta\phi} - M_{h\zeta})M_{h\zeta} \right) H_0^2 + \frac{1}{96} \left(24M_{h\zeta}^2 M_\phi + 90M_f^2 M_\zeta + 24M_{h\phi}^2 M_\zeta - 24M_{h\phi} M_{h\zeta} M_{\zeta\phi} - (8M_1 - 3M_g + 24M_h)(4M_\phi M_\zeta - M_{\zeta\phi}^2) + 6M_f(-6M_{h\zeta}^2 + (8M_1 - 3M_g + 24M_h + 12M_{h\phi} - 4M_\phi)M_\zeta - 6M_{h\zeta} M_{\zeta\phi} + M_{\zeta\phi}^2) \right), \quad (61)$$

$$\mathcal{R} = \psi + t^{5/2} \left(C_3 t^{\frac{\sqrt{9A^2 H_0^2 - 32AB}}{2AH_0}} + C_4 t^{\frac{-\sqrt{9A^2 H_0^2 - 32AB}}{2AH_0}} \right), \quad (62)$$

where C_3 and C_4 are some functions of C_1 , C_2 and M_i . To have a decaying mode for the curvature perturbation, one should keep the positive power of time only and this leads to

$$\alpha_3 \leq 0 \quad \& \quad \alpha_4 < 0 \quad \text{or} \quad \frac{1}{2} \alpha_3^2 < \alpha_4 < \frac{9}{16} \alpha_3^2 \quad \left(\text{for } -1 < \alpha_3 < -\frac{2}{3} \text{ we have } \alpha_4 \neq \frac{1}{4}(1 + 3\alpha_3) \right), \quad (63a)$$

$$\alpha_3 > 0 \quad \& \quad \alpha_4 < \frac{9}{16} \alpha_3^2, \quad \alpha_4 \neq 0. \quad (63b)$$

V. CONCLUSIONS AND FINAL REMARKS

In this paper we studied the cosmological perturbations of the Quasi-Dilaton massive gravity. This theory is the extension of the non-linear massive gravity theory recently proposed by de Rham, Gabadadze and Tolley through a scalar field. The scalar field is coupled to the mass term in such a way that the field space of the theory admits a dilatation invariance. This new symmetry of the theory enables us to obtain flat FRW solutions. If considered without matter, the theory predicts an accelerating solution which is the effect of the graviton mass. The stability of this solutions is considered in [23] where the authors find that the ω parameter has to have a positive value less than 6, and the range for the parameter α_4 has to be less than $\alpha_3^2/8$.

Our analysis of cosmological perturbations shows that the vector mode vanishes when one uses the gauge invariant variables. The tensor mode has a different behavior to that of the standard GR but similar to the gravitational waves obtained in the dRGT massive gravity theory. The gravitational waves in this theory have a non-vanishing time-dependent mass which modifies the dispersion relation of the gravitational waves.

In order to find the scalar spectrum of the theory, one can use the gauge invariant variables and then integrate out two of the non-dynamical variables included in the metric perturbation. The equations of motion of the remaining three scalar perturbations can then be obtained by varying the resulting action. As a matter of fact, the procedure is so difficult that one cannot solve the equations analytically. However, as long as we are interested in studying the behavior of the theory at the superhorizon scales, we can study the lagrangian over that scales. One of the scalar perturbations does not play any role in the superhorizon scales because it always comes with a wave number. The resulting superhorizon Lagrangian can then be varied with respect to ψ and the curvature perturbation \mathcal{R} . The equations can be solved analytically if one assumes that the quasi-dilaton field has no kinetic term. We can then obtain conditions for which the curvature perturbation is damped over the superhorizon scales, i. e. equation (63).

However, if one considers the range for α_4 where the solution is stable (equation (7)), one may reduce the allowed parameter space to be

$$\alpha_3 > 0 \quad \text{and} \quad 0 < \alpha_4 < \frac{\alpha_3^2}{8}, \quad (64)$$

for $\omega = 0$. One may note that the above range for the parameter space can be different if one considers a non-zero ω parameter.

Finally, the QD massive gravity theory has a potential of producing a reasonable inflationary scenario, if one adds a potential to the action or change the graviton mass to be a function of the quasi-dilaton field. This can then break the dilatation invariance, which we will study in future works.

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Appendix A: Constants of the second order mass Lagrangian

$$M_1 = \frac{m^2}{128\alpha_4^2} \left[9r\alpha_3^2(-1 + 3(X-1)\alpha_3) + 4(1 + 3r - 3(-3 + 4r(X-1) + X)\alpha_3 - 18(X-1)\alpha_3^2)\alpha_4 \right. \\ \left. - 16(8 + 9\alpha_3 - X(7 + 6\alpha_3) + 3r(-2 + X - 4\alpha_3 + 3X\alpha_3))\alpha_4^2 - 192(r-1)(X-1)\alpha_4^3 \right], \quad (A1)$$

$$M_2 = \frac{m^2}{32\alpha_4} \left[r(1 + 6\alpha_3)(3(X-1)\alpha_3 - 1) + 4(6 - 4r - 3X - rX + 3(4 - 3X + r(4X - 5))\alpha_3\alpha_4 + 48(r-1)(X-1)\alpha_4^2) \right], \quad (A2)$$

$$M_\zeta = \frac{1}{32\alpha_4^2} 3m^2 \left[9(7r-3)\alpha_3^2(3(X-1)\alpha_3 - 1) + 4(-4 + 12r + 3(3+r+7X-19rX)\alpha_3 + 18(9r-5)(X-1)\alpha_3^2)\alpha_4 \right. \\ \left. + 16(2 + X(14 - 33\alpha_3) + 39\alpha_3 + r(6 - 51\alpha_3 + 15X(3\alpha_3 - 2)))\alpha_4^2 + 384(r-1)(X-1)\alpha_4^3 \right], \quad (A3)$$

$$M_\phi = \frac{3m^2}{32\alpha_4^2} [-3\alpha_3^2 + 9(X-1)\alpha_3^3 + 4\alpha_4 - 16(X-1)\alpha_3\alpha_4], \quad (A4)$$

$$M_{h\zeta} = \frac{m^2}{32\alpha_4^2} \left[27r\alpha_3^2(1 - 3(X-1)\alpha_3) - 4(2 + 4r - 3(-6 + 3r + 2X + 7rX)\alpha_3 + 18(-2 + 5r)(X-1)\alpha_3^2)\alpha_4 \right. \\ \left. - 16(8 + X(2 - 24\alpha_3) + 30\alpha_3 + r(-2 - 39\alpha_3 + X(-14 + 33\alpha_3)))\alpha_4^2 - 384(r-1)(X-1)\alpha_4^3 \right], \quad (A5)$$

$$M_{\zeta\phi} = \frac{m^2}{16\alpha_4^2} \left[27(2(r-1)r-1)\alpha_3^2(3(X-1)\alpha_3 - 1) + 4\left(-6 + 8(r-1)r + 3(9(X-1) + 2r(2+r+7X-7rX))\alpha_3 \right. \right. \\ \left. \left. + 54r(2r-3)(X-1)\alpha_3^2\right)\alpha_4 + 16(4X - 6 + 8r(r-1+3X-2rX) + 9(1+2(r-2)r)(X-1)\alpha_3)\alpha_4^2 \right], \quad (A6)$$

$$M_{h\phi} = \frac{m^2}{8\alpha_4} \left[(1 + 6\alpha_3)(3(X-1)\alpha_3 - 1) + 4(2 - 3\alpha_3 + X(-4 + 3\alpha_3))\alpha_4 \right], \quad (A7)$$

$$\begin{aligned}
M_\beta = & -\frac{m^2}{32r_1\alpha_4^2} \left[9\alpha_3^2 \left(r_1 + 6rr_2 - 2(3+4r)r_3 + (8+3r)r_4 - 3r_5 \right) \left(3(X-1)\alpha_3 - 1 \right) \right. \\
& + 4 \left\{ 8rr_2 - (8+9r)r_3 + 3(3+r)r_4 - 3r_5 + 3 \left[r_1 + \left(-4(X-1)r_1 - 2r(3+7X)r_2 + 6r_3 + 13rr_3 + 14Xr_3 \right. \right. \right. \\
& + 17rXr_3 - 13r_4 - 6rr_4 - 17Xr_4 - 6rXr_4 + 6(1+X)r_5 \left. \right) \alpha_3 + 6(X-1)[10rr_2 - 5(2+3r)r_3 \\
& + 3(5+2r)r_4 - 6r_5] \alpha_3^2 \left. \right\} \alpha_4 + 16 \left\{ rr_2[-4-25X+(75X-78)\alpha_3] + r_3[4+12r+25X+33rX \right. \\
& + 3(26+43r-25X-42rX)\alpha_3] + 6r_5[1+2X-9(X-1)\alpha_3] \\
& + 3r_4 \left(-4-2r-11X-4rX + [-43+18r(X-1)+42X]\alpha_3 \right) \left. \right\} \alpha_4^2 \\
& \left. + 192[r(5X-4)r_2 + (4+7r-5X-9rX)r_3 + (-7-3r+9X+4rX)r_4 + (3-4X)r_5] \alpha_4^3 \right]. \tag{A8}
\end{aligned}$$

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- [1] M. Fierz, W. Pauli, Proc. Roy. Soc. Lond. A173, 211-232 (1939).
[2] H. van Dam, M. J. G. Veltman, Nucl. Phys. B22, 397-411 (1970); V. I. Zakharov, JETP Lett. 12, 312 (1970).
[3] A. I. Vainshtein, Phys. Lett. B 39, 393 (1972).
[4] D. G. Boulware, S. Deser, Phys. Rev. D6, 3368-3382 (1972).
[5] N. Arkani-Hamed, H. Georgi, M. D. Schwartz, Annals Phys. 305, 96-118 (2003).
[6] P. Creminelli, A. Nicolis, M. Papucci, E. Trincherini, JHEP 0509, 003 (2005).
[7] C. de Rham, G. Gabadadze, A. J. Tolley, Phys. Rev. Lett. 106, 231101 (2011).
[8] C. de Rham, G. Gabadadze, Phys. Rev. D82, 044020 (2010).
[9] M. Mirbabayi, Phys. Rev. D 86, 084006 (2012).
[10] G. D'Amico, C. de Rham, S. Dubovsky, G. Gabadadze, D. Pirtskhalava, A. J. Tolley, [arXiv:1108.5231].
[11] A. E. Gumrukcuoglu, C. Lin and S. Mukohyama, JCAP 1111, 030 (2011), [arXiv:1109.3845].
[12] A. Emir Gumrukcuoglu, Chunshan Lin and Shinji Mukohyama, JCAP 03 (2012) 006, arXiv:1111.4107v2 [hep-th].
[13] P. Gratia, W. Hu and M. Wyman, Self-accelerating Massive Gravity: Exact solutions for any isotropic matter distribution, Phys. Rev. D 86, 061504 (2012), [arXiv:1205.4241]; G. D'Amico, Phys. Rev. D 86, 124019 (2012), [arXiv:1206.3617]; M. Fasiello and A. J. Tolley, JCAP 1211, 035 (2012), [arXiv:1206.3852]; Y. Gong, [arXiv:1207.2726]; M. S. Volkov, Phys. Rev. D 86, 104022 (2012), [arXiv:1207.3723]; C. -I Chiang, K. Izumi and P. Chen, JCAP 1212, 025 (2012), [arXiv:1208.1222]; H. Motohashi and T. Suyama, Phys. Rev. D 86, 081502 (2012), [arXiv:1208.3019]; D. Langlois and A. Naruko, [arXiv:1206.6810].
[14] K. Hinterbichler, Rev. Mod. Phys. 84, 671 (2012), [arXiv:1105.3735].
[15] S. F. Hassan and R. A. Rosen, JHEP 1202, 126 (2012), [arXiv:1109.3515].
[16] S. F. Hassan and R. A. Rosen, JHEP 1204, 123 (2012), [arXiv:1111.2070].
[17] N. Khosravi, H. R. Sepangi and S. Shahidi, Phys. Rev. D 86, 043517 (2012); D. Comelli, M. Crisostomi, F. Nesti and L. Pilo, JHEP 1203, 067 (2012) [Erratum-ibid. 1206, 020 (2012)], [arXiv:1111.1983]; M. S. Volkov, JHEP 1201, 035 (2012), [arXiv:1110.6153]; M. von Strauss, A. Schmidt-May, J. Enander, E. Mortsell and S. F. Hassan, JCAP 1203, 042 (2012), [arXiv:1111.1655]; M. Crisostomi, D. Comelli and L. Pilo, JHEP 1206, 085 (2012), [arXiv:1202.1986]; M. S. Volkov, Phys. Rev. D 86, 061502 (2012), [arXiv:1205.5713]; S. i. Nojiri and S. D. Odintsov, Phys. Lett. B 716, 377 (2012), [arXiv:1207.5106];
[18] K. Nomura and J. Soda, Phys. Rev. D 86, 084052 (2012).
[19] K. Hinterbichler, R. A. Rosen, JHEP 07 (2012) 047.
[20] N. Khosravi, N. Rahmanpour, H. R. Sepangi and S. Shahidi, Phys. Rev. D 85, 024049 (2012), [arXiv:1111.5346].
[21] Q. -G. Huang, Y. -S. Piao and S. -Y. Zhou, Phys. Rev. D 86, 124014 (2012), [arXiv:1206.5678].
[22] E. N. Saridakis, [arXiv:1207.1800]; Y. -F. Cai, C. Gao and E. N. Saridakis, JCAP 1210, 048 (2012), [arXiv:1207.3786]; G. Leon, J. Saavedra, E. N. Saridakis, arXiv:1301.7419 [astro-ph.CO]; K. Hinterbichler, J. Stokes and M. Trodden, [arXiv:1301.4993]; D. -J. Wu, Y. -F. Cai and Y. -S. Piao, [arXiv:1301.4326].
[23] G. D'Amico, G. Gabadadze, L. Hui and D. Pirtskhalava, [arXiv:1206.4253].